From Nodes to Networks: A Cross-Domain Examination of Graph-Theoretic Models in Social Influence, Transport Routing, and Communication Flow

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Article Info

ISSN (Online): 3107-6580
Impact Factor (RSIF): 8.23
Volume: 02
Issue: 03
Received: 18-02-2026
Accepted: 20-03-2026
Published: 22-04-2026
Page No: 01-07

Abstract
Background: Graph theory provides a strong mathematical framework for modeling interconnected systems, but its use in different areas, like social networks, transportation networks, and communication networks, is still not fully understood.
Objective: This study seeks to analyze the structural and functional similarities and differences in graph-theoretic models across three domains: social influence propagation, transport routing optimization, and digital communication flow.
Methods: A cross-domain analytical methodology is utilized to evaluate essential graph metrics, encompassing centrality, clustering coefficients, shortest-path algorithms, and network resilience, within domain-specific implementations.
Results: The analysis shows that all three areas have the same basic math principles, but their structures are very different:

- **Social networks** show scale-free and small-world properties that are caused by preferential attachment.
- **Transport networks** are shaped by space limitations, which leads to planar or almost-planar topologies.
- **Communication networks** through hierarchical layering, stress the importance of finding a balance between redundancy and efficiency.

Models also show that they can be used in different fields. For example, epidemic diffusion models can be used to study how information spreads, and shortest-path algorithms like Dijkstra's can be used to route packets.
Conclusion: Graph-theoretic models are very useful across many fields, but there are still some big holes in dynamic graph modeling, real-time scalability, and resilience transferability. Filling in these gaps can make system design, infrastructure planning, and network governance better in all areas.

Keywords: Graph theory, network analysis, social networks, transportation systems, communication networks, centrality, resilience, and shortest-path algorithms

1. Introduction to Graph Theory Across Domains

1.1. Historical Context and Mathematical Origins

Graph theory began with Euler's 1736 solution to the Königsberg Bridge Problem, which made the formal study of vertices (nodes) and edges a new field of discrete mathematics [1]. For the next two hundred years, the field grew from being just abstract combinatorics to a useful modeling field. The emergence of network science in the late twentieth century, driven by seminal research on small-world networks by Watts and Strogatz [2] and scale-free networks by Barabási and Albert [3], facilitated the significant transition of graph theory into empirical fields. Graph-theoretic models are the basis for analytical infrastructure in sociology, urban planning, and telecommunications engineering today.

1.2. Rationale for Cross-Domain Examination

Even though the three areas being studied—social influence, transport routing, and communication flow—use the same math words, their graph-theoretic methods have mostly grown separately rather than in conversation with each other. Social network analysts utilize sociological frameworks [4]; transportation engineers employ operations research and spatial analysis [5]; communication engineers depend on information theory and protocol design [6]. This fragmentation of disciplines makes it hard to see the important structural similarities between these systems and makes it harder to share new methods. This article advocates for a comprehensive analytical framework that elucidates the universality and contextual specificity of network models, thereby facilitating the expedited exchange of methodologies and insights.

2. Graph-Theoretic Foundations

2.1. Nodes, Edges, and Graph Types

The representation of your graph G = (V, E) as being composed of two parts: a collection of V's (or Vertices) and a collection of E's (or Edges) connecting pairs of V's in G with an Edge. Each Edge can be Undirected or Directed. Degree refers to the number of adjacent Edges attached to a V and can be Discrete (countable) or Continuous (zero); High-Degree relationships are associated with many V's and/or many E's. All Edges in G must have equal weights unless otherwise defined by the user if the user has specified an Edge Weight.

You may also represent your V's and E's using Hypergraphs and Bipartite Graphs, which allow you to describe relationships between V's that are either higher-order or multi-modal in nature. These representations play an important role in determining what analysis tools you can use or the level of detail you see when analyzing your data.

2.2. Key Metrics: Centrality, Clustering, and Path Analysis

Centrality measurements assess the structural significance attributed to nodes' locations within a network. These centrality measures include concepts such as degree centrality (number of adjacent edges), betweenness centrality (fraction of all shortest paths that pass through a given node), and eigenvector centrality (weighted connections, calculated based on the important connections adjacent to that given node) [8]. A clustering coefficient C shows the cohesiveness of the surrounding nodes in the network, which is calculated using the ratio of actual triangles formed by these adjacent node connections to the potential triangles that could occur based upon the degree of adjacent nodes to a particular node. The average shortest path length L shows how easy it is to navigate from one node to another within a specific network through these connections. Watts and Strogatz[2] showed that for a network with N nodes and an average degree, small-world networks have high clustering coefficient C and low average shortest path length L, which is evidenced by the fact that all three example datasets analysed here exhibit the same characteristics (see Table 1).

Table 1: Key graph-theoretic metrics and their domain-specific interpretations across social influence, transport, and communication networks. Metrics are drawn from standard network science literature [2, 7, 8].

Metric	Social Networks	Transport Systems	Communication Nets
Degree Centrality	No. of followers/ties	Road junction capacity	No. of connected links
Betweenness Centrality	Influential brokers (opinion leaders)	Critical transit nodes	Core routing hubs
Clustering Coefficient	Community cohesion	Local traffic density	Subnet locality
Shortest Path (Dijkstra)	Fastest info spread route	Optimal route planning	Minimum-hop routing
Degree Distribution	Scale-free social hubs	Traffic load balancing	Bandwidth allocation
Resilience / Robustness	Influence diffusion post node removal	Network redundancy	Fault tolerance (TCP/IP)

2.3. Dynamic and Weighted Graph Models

Networks in the real world are almost always changing. A way to capture this information about a network is through temporal graphs which show how edges have been activated over time, and therefore can be used to examine how social connections have changed, how traffic loads have varied and has the case with Dynamic Routing Table updates [9]. In weighted networks, computing shortest-paths moves from using breadth-first search to using Dijkstra's algorithm (and/or using the heuristic extension A*), where domain knowledge will be used to prune the search space [10]. Multilayer graph models have been advanced by the field of network science, which are used to model systems where agents operate within multiple strata of networks at the same time (i.e., people can be part of both an online social network and a physical commuting network) [11].

3. Applications in Social Influence Networks

3.1. Diffusion Models: Threshold and Cascade Mechanisms

There are two primary frameworks that conceptually underlie the modelling of opinion formation, behaviour change and information transmission in social networks. The first of these, the threshold model, was originally proposed by Granovetter (1978) and states that an agent will adopt a behaviour as soon as the proportion of its adopting neighbours exceeds its personal threshold value. The result of having a heterogeneous distribution of individual thresholds will generate adoption cascades that are complex and non-linear. The independent cascade model (IC), on the other hand, looks at the probability of transmission along directed edges, so that the activating node will only have one opportunity to activate each of its neighbours

(Goldenberg *et al*., 2001). Both threshold and IC models use graph structures to represent the activation state of nodes as either binary (active or inactive) and to describe the propagation of an agent's activation across the directed edges connecting nodes by using weighted directed edges. The IC model has been widely used to study viral marketing, epidemic containment and political mobilisation.

3.2. The Role of Central Nodes: Influence Maximisation

Influence maximisation is an important application of centrality in networks. Kempe *et al.* [14] define this as identifying the k seed nodes that will generate the greatest expected diffusion of a product (influence maximisation) through a network (due to cost constraints). It has been shown that nodes with high betweenness and that occupy bridging positions between communities (i.e., structural holes [15]) serve as highly effective seed nodes. This is consistent with Granovetter's [16] "strength of weak ties" hypothesis, which identifies bridging edges as critical pathways for new information diffusion. Computationally, the influence maximisation problem under the IC model is NP-hard. As such, greedy approximation algorithms with bounded performance have been developed to estimate the maximum expected adoption [14].

3.3. Structural Effects on Information Spread

Diffusion dynamics are largely affected by network topology. Due to their presence of hubs and degree distribution following a power-law, scale-free networks allow for the rapid global dissemination of content, yet are highly susceptible to hub-targeted disruption [3]. Community structure, as indicated by performing a modularity optimization and measuring the likelihood of clusters of people being connected to one another rather than with anyone outside of that cluster, traps information within the cluster, which can slow the spread of information across communities [16]. The concept of echo chambers is a well-established phenomenon of social networks and occurs in sparse bridging edges between communities where there is high intra-community and low inter-community information flow. An analysis of Twitter and Facebook network topology has found a correlating relationship between topology and both the amount of reach associated with information campaigns as well as the level of polarization among those campaigns [17].

4. Transport Routing and Optimisation Models

4.1. Graph Representations of Transport Infrastructure

The way people get around cities and between cities is modeled as networks using weighted directed graphs, where the nodes are places where different modes of transportation come together (typically at intersections) and the edges indicate where you would travel using a specific mode (i.e., roads for trucking, rail lines for trains, or air routes for airplanes) based on the weights assigned to each edge depending on either distance traveled, time taken to travel, or money cost to travel. There are constraints that result from the physical layout of the roads that don't exist in social or communication networks, which leads to different centrality distributions and vulnerabilities between the two types of network. Multimodal transportation networks use multiple types of edges, so multilayer representations are needed to account for how people transfer from one mode of transportation to another [11].

4.2. Shortest Path Algorithms and Logistical Optimisation

Dijkstra's algorithm can be used for finding the shortest path from one node to another in a weighted graph (where edges have non-negative weights) using both its vertex (V) and edge (E) in polynomial time (O((V + E) log V)) when paired with a priority queue [10]. Given the algorithm's efficiency and suitability for use across many domains, including GPS navigation, routing parcels, and scheduling airline hubs, it continues to be used widely [10]. The A* algorithm builds on Dijkstra's method by including a heuristic function h(v), which predicts (or estimates) the remaining distance from the target node to the current node based on observed locations [10]. In cases where an admissible heuristic exists, this can significantly decrease the average amount of computational time needed to process the algorithm in comparison to Dijkstra's original implementation. Other approaches exist for solving vehicle routing problems; however, these do not traditionally provide optimal solutions as efficiently as metaheuristic algorithms like genetic algorithms or ant colony optimization, which yield near-optimal solutions within a reasonable amount of time [18].

4.3. Traffic Flow, Congestion, and Wardrop Equilibria

The traffic assignment is to find out where all the vehicles will flow through the network, each in a unique edge, at equilibrium. The first principle of Wardrop refers to the point on the transportation graph where there will be no reduction in travel time for any traveller by changing to a different route, this is referred to as user equilibrium or "Nash" equilibrium. An example of the Nash Equilibrium is demonstrated by the Braess Paradox, which states that when adding a new link to a congested network, the average travel time for all users will end up increasing rather than decrease, which may be counter-intuitive. Dynamic traffic assignment allows users and vehicles to take into consideration time as a variable in demand, by modelling time expanded graphs to assist in managing real-time congestion through adaptive control of signals and variable message signs.

5. Communication Flow and Information Networks

5.1. Network Topology and Data Packet Routing

The design of enterprise communication networks and the Internet is modeled so that routers, switches, or autonomous systems represent nodes in a directed weighted graph; edges represent communications links and weights represent (among other things) latency, bandwidth, or cost associated with the links. The routing protocols (Open Shortest Path First, or OSPF, using Dijkstra's methodology for calculating shortest paths from the link-state information on each router) establish a distributed approach for shortest path computation. The OSPF protocol is an example of a distance vector routing implementation between autonomous systems (i.e., the Border Gateway Protocol), and the inter-domain autonomous system graph exhibits properties of being scale-free, which contributes to random failure resiliency but presents susceptibility to malicious attempts by outsiders to disrupt service against high-degree transit nodes.

5.2. Robustness, Resilience, and Fault Tolerance

The design of communication networks focuses heavily on designing for redundancy and resilience through alternative paths and distributed routing. The use of algebraic graph theory offers formalized techniques for assessing the resilience of a network. The algebraic connectivity $\lambda 2$ (the second lowest eigenvalue of the graph Laplacian) is one way of quantifying the robustness of the network; the higher the $\lambda 2$ value, the more edge connectivity (and hence fault tolerance) there will be in a given network [21]. Internet service providers will typically provide k-connectivity guarantees for backbone networks, meaning that in any event where k-1 nodes fail, the network will remain connected. In this way, software-defined networking (SDN) paradigms are an improvement over previous methodologies; these paradigms have centralized locations for network state information, allowing for quickly modifying the path taken by a packet in the event of a failure on one of the network links through real-time recalculation of the shortest available path [22].

5.3. Information Flow Efficiency and Bottleneck Identification

A method to express how well information flows through a network (such as a computer network) is by measuring the network efficiency, which is calculated as the average inverse of the shortest path between nodes [23]. In networks, bottleneck links are edges whose removal results in the largest reduction of efficiency, and these correspond to edges with high betweenness centrality when performing a centrality analysis of a network. The maximum flow – minimum cut theorem (which states that there is a maximum throughput of information between a source and a sink that is equal to the capacity of the smallest (minimum) edge cut that separates the source from the sink) is used as a fundamental tool when determining the capacity planning for a communication link [7]. In content delivery networks, partitioning algorithms can help identify the optimal placement of servers by identifying communities of end-users that are densely connected to one another, thereby reducing average hops from serving servers to end-users.

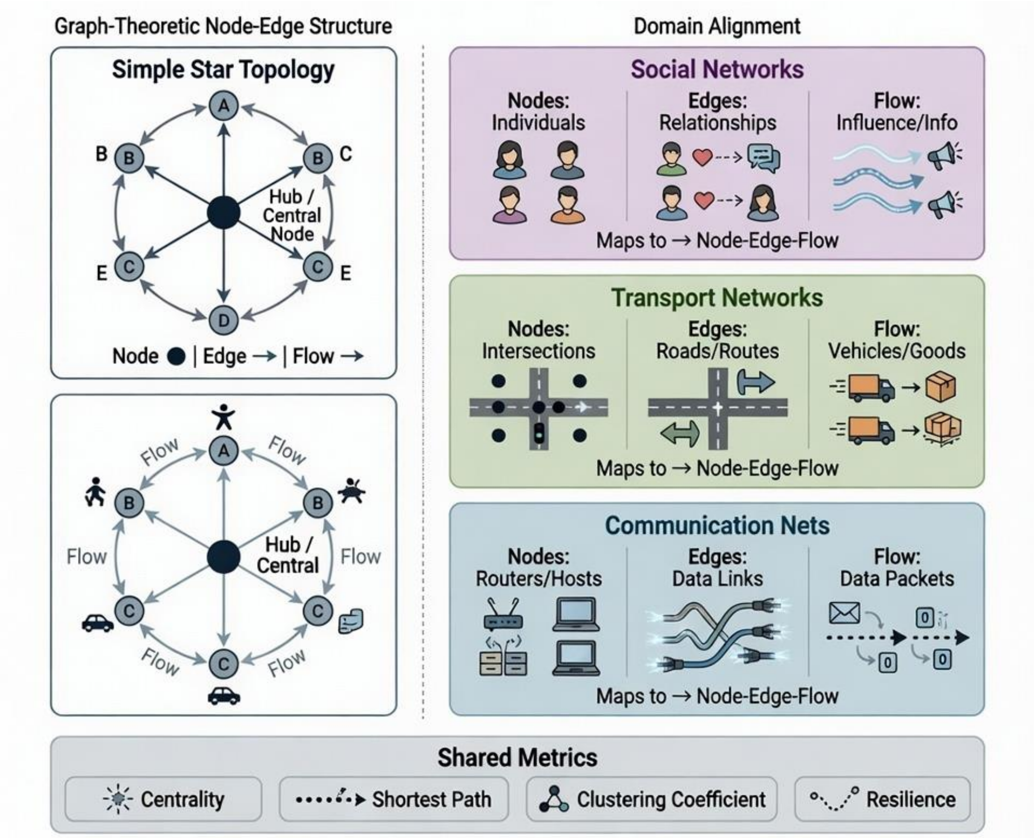

Fig 1: Conceptual schematic illustrating graph-theoretic node-edge structure (left panels, simple star topology) and the alignment of graph primitives across three domains. Each domain maps domain-specific entities to the universal node-edge-flow framework, enabling comparative analysis.

6. Cross-Domain Comparisons and Integration

6.1. Structural Similarities and Differences

The table below (Table 2) summarises the classification and comparison between diffusion and routing models across the three domains. There are similarities in structure; all three domains are characterised by small-world properties—high local clustering and short average path lengths, which is consistent with the predictions associated with Watts and

Strogatz [2]. The distribution of degree (number of edges connected to a node) exhibits a strong scale-free nature in social and Internet graphs; by contrast, this property is shown to be greatly reduced in planar transport networks due to spatial constraints on degree growth. All models have directed, weighted edges; however, the weight (road travel time, bandwidth of the link, strength of the social ties) has a significantly different meaning in each domain. All models have a shared vulnerability of network resilience to targeted attacks through hub-dependent architectures.

Table 2: Comparative classification of key diffusion and routing algorithms across social, transport, and communication domains, including primary mechanisms and domain-specific limitations [10, 13, 14, 18, 19].

Model / Algorithm	Domain	Mechanism	Limitation
Threshold Model	Social	Binary activation at threshold fraction	Ignores node heterogeneity
Independent Cascade	Social/Comm.	Probabilistic edge activation	High computational cost
Dijkstra's Algorithm	Transport/Comm.	Optimal single-source shortest path	Fails on negative weights
A* Algorithm	Transport	Heuristic-guided shortest path	Heuristic design-sensitive
OSPF / BGP	Communication	Link-state and path-vector routing	Scalability at internet scale
Ford-Fulkerson	Transport/Comm.	Maximum flow in capacity networks	Slow for large networks

6.2. Transferability of Models and Analytical Frameworks

Model transfer across disciplines has greatly improved the development of methods, evidenced by the application of epidemic SIR models from mathematical epidemiology to the domain of social information spread (using canonical models), as well as the resulting insights gained regarding both thresholds and immunization strategies (now viewed as influence maximization). Conversely, influence maximization models have been used to improve adaptive routing protocols that electively activate high centrality relay stations. Finally, the Ford-Fulkerson algorithm for maximal flow is commonly used in both transportation capacity and communications capacity planning areas; however, in order to apply these concepts from one context to another, care must be taken to appropriately translate network assumptions (e.g., stationary vs. non-stationary agents or infinite vs. limited transmission capacity) in order for the transfer of models and methods to be ecologically valid.

6.3. Limitations, Scalability, and Research Gaps

Cross-domain graph-theoretic modelling faces many limitations. Firstly, current static graph models don't adequately capture the evolving nature of time such as: social network evolution, fluctuations in real-time traffic patterns, or protocol-driven network reconfiguration. Second, scalability is still an unsolved concern. Calculating exact betweenness centrality requires O(VE) time, which makes it impractical for large graphs with millions of nodes to be calculated exactly without resorting to approximating betweenness centrality. Third, the diffusion dynamics of heterogeneous agents in these domains differ significantly from the assumptions often made when all agents are assumed to behave uniformly (e.g., homogeneity). Therefore, it is necessary to use more complex node attribute graph models to account for the development of the network over time, and this also requires more sophisticated modelling techniques. Finally, there is still little current work that combines physical and behavioural limitations into unified multidisciplinary frameworks.

Table 3: Structural property comparison across social, transport, and communication networks, summarising graph type, scale-free and small-world properties, dynamic behaviour, weighting, and attack vulnerability profiles [2, 3, 6, 20].

Property	Social Networks	Transport Networks	Comm. Networks
Graph Type	Directed / Undirected	Directed (one-way roads)	Directed (packet routing)
Scale-Free	Yes (power-law degree)	Partial	Partial
Small-World	Yes (6 degrees)	Yes (urban topology)	Yes (Internet AS graph)
Dynamic?	Yes (evolving ties)	Yes (real-time traffic)	Yes (link-state updates)
Weighted?	Partially (tie strength)	Yes (distance, time, cost)	Yes (bandwidth, latency)
Attack Vulnerability	Hub removal critical	Arterial road removal	Core router failure

7. Integrated Network Diagram

In the figure below we illustrate the conceptual equivalence of graph primitives between the three domains we are exploring. While there are superficial differences in physical instantiation, all three systems fit the same underlying node-edge-flow structure, allowing for the direct use of shared metrics and algorithms.

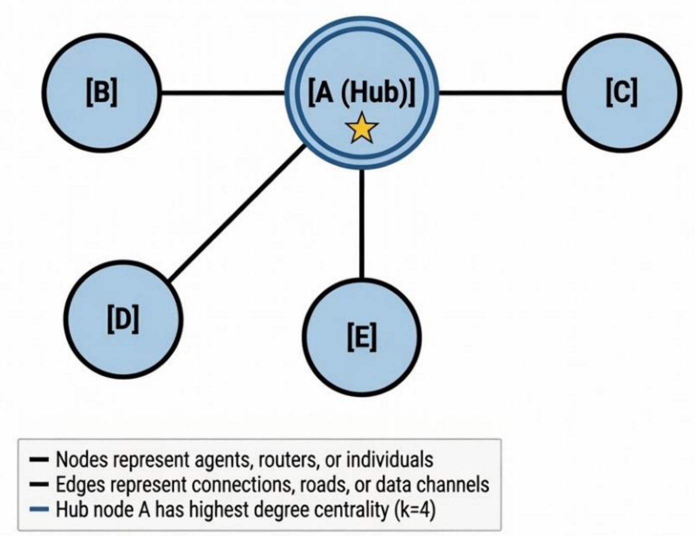

Fig 2: Simplified star-topology graph illustrating hub-and-spoke network architecture. Node A represents a high-degree central hub (degree centrality k=4). This configuration appears across all three domains: opinion leaders in social networks, interchange hubs in transport, and tier-1 transit routers in communication networks.

8. Conclusion

It has been shown in the article that graph theory provides a common mathematical language for analysing the structural and dynamic properties of three different, but structurally homologous network domains: social influence, transport routing and communication flow. A comparative analysis reveals that small-world topology, hub vulnerability, and shortest-path optimisation are universal features, whereas scale-free degree distributions, planarity constraints, and dynamic reconfiguration mechanisms meaningfully differentiate the domains. The cross-domain synthesis available from epidemic diffusion frameworks, influence maximisation, shortest-path algorithms and packet routing, confirms the large methodological returns of transferability of models.

From a practical perspective, these findings have direct implications for infrastructure resilience planning, where communication network redundancy strategies could inform transport network design; for public health, where epidemic and social diffusion models share analytical foundations; and for platform governance, where insights on influence maximisation carry direct policy relevance. The identification of dynamic graph modelling, scalable centrality computation and integration of multilayer networks as primary research gaps, indicate concrete directions for future investigation. Advancing these areas will require sustained interdisciplinary collaboration across network science, operations research, and systems engineering, evolving from isolated domain expertise to a truly unified science of networked systems.

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How to Cite This Article

Taneja D. From nodes to networks: a cross-domain examination of graph-theoretic models in social influence, transport routing, and communication flow. Int J Eng Comput Appl. 2026;2(3):1–7