International Journal of Engineering and Computational Applications

A Generalized Bianchi Type I Cosmological Model with Time-Varying Gravitational and Cosmological Constants in f (R, T) Gravity

Saeed Ahmad
Department of Mathematics and Statistics, The University of Lahore, Lahore, Pakistan

* Corresponding Author: **Saeed Ahmad**

Article Info

ISSN (Online): 3107-6580
Impact Factor (RSIF): 8.23
Volume: 02
Issue: 04
Received: 25-04-2026
Accepted: 27-05-2026
Published: 29-06-2026
Page No: 27-31

Abstract
The main aim of this paper is an extension of the cosmological model with variable gravitational constant G and cosmological constant Λ which has been developed within the framework of f(R,T)) gravity. The study is motivated by earlier investigations of anisotropic Bianchi Type I cosmological models with varying G and Λ in General Relativity. The present work generalizes these models by considering a homogeneous Bianchi Type I space-time filled with a perfect fluid and incorporating variable G and Λ in modified gravity. The analysis focuses on the specific form f(R,T)=R+2f(T)) where the dependence on the trace of the energy-momentum tensor introduces modifications to Einstein's field equations. Exact solutions of the modified field equations have been obtained for an anisotropic Bianchi Type I universe with a time-dependent cosmological term. The model naturally describes both the early inflationary phase and the present accelerated expansion of the universe, providing a unified cosmological scenario. The physical and geometrical behavior of the model has been examined through various cosmological parameters. The solutions reveal a coupling among the shear scalar σ^2 gravitational constant G, cosmological constant Λ and Hubble parameter H, leading to a linear relationship between these quantities. The cosmological term is assumed to vary as $\Lambda \propto R^{-2}$ where R is the scale factor. The obtained model exhibits a point-type singularity at the initial epoch. At this stage, the cosmological constant becomes infinitely large, while the gravitational constant decreases with cosmic evolution. As time progresses, Λ gradually decreases, consistent with current cosmological observations. No specific equation of state or predefined functional form of G has been imposed in the analysis. Furthermore, the anisotropy decreases with time, and the universe evolves toward isotropy at late times, satisfying the condition $\sigma \to 0$. Therefore, the model successfully describes a quasi-isotropic universe and provides a viable framework for studying cosmological evolution in modified f(R,T)) gravity with variable gravitational and cosmological parameters.

DOI: https://doi.org/10.54660/.IJECA.2026.2.4.27-31

Keywords: Modified f(R,T) Gravity, Bianchi Type I Cosmological Model, Cosmological constant (Λ), Gravitational Constant (G), Anisitropic Universe, Cosmological Evalution

Introduction

General Relativity (GR) was introduced by Einstein in 1915 and completely changed our view on gravitation and cosmology at large [1]. Gravity is understood by GR as curving space-time caused by the presence of matter and energy, GR says. General Relativity has been very successful at describing a variety of astrophysical and cosmological phenomena, but some observations are now becoming a serious problem for the standard cosmological model [2,3], notably the accelerating expansion of the universe.

A distant-type Ia supernova [2–3] and the Cosmic Microwave Background (CMB) radiation [4] observations have revealed that the present universe is expanding at an accelerated rate. As part of Einstein's theory, this acceleration can be understood by adding the cosmological constant Λ, a source of dark energy. The cosmological constant has, however, very severe theoretical problems, including the fine tuning problem and the coincidence problem. These problems have sparked cosmologists to look for alternate solutions, such as modified theories of gravity [5].

One of the best investigated modifications of GR is f(R) gravity. According to this theory, the Einstein-Hilbert action is generalized replacing the Ricci scalar R with an arbitrary function f(R) [6, 7]. These changes inevitably emerge in an attempt to quantum correct gravity, and it has been demonstrated that they can give a geometrical explanation for the early inflationary phase, as well as the late-time accelerated expansion of the universe, without the need of exotic dark energy components [6]. In modern theoretical cosmology, several cosmological models via f(R) gravity have been successfully used during various phases of evolution of the universe, and have gained great interest [7].

Another generalization of f(R) gravity was introduced by Harko *et al.* [8] in the form of the f(R,T) gravity, which involves the action depending on the scalar R and the trace T of the energy-momentum tensor. The explicit coupling to T, on the other hand, couples the matter to the geometry, resulting in changes to the gravitational field equations. The coupling of this matter with geometry can lead to further gravitational effects and to new understanding regarding the accelerated expansion of the Universe, dark energy effects and cosmological dynamics [8]. The richness of the mathematical structure and physical meaning of f(R,T) gravity has made it an interesting field of research in the modern cosmology.

Interestingly, it is possible in cosmology that the fundamental constants in nature can also change over time. Dirac proposed the Large Number Hypothesis in [9] which leads to the idea of a varying gravitational constant, G. Many investigations have dealt with the possible cosmological models involving variable G and Λ in order to comprehend their effect on cosmic evolution since then. A time-dependent cosmological constant (C) can provide a natural explanation for the present accelerated expansion, and a varying gravitational constant (C) can have an impact on the dynamics of the universe from the early epochs to the present day [10].

Anisotropic models of the cosmos are especially useful in the study of the early universe. While the large-scale universe is highly isotropic, anisotropies might have been important in the early universe. Bianchi Type I space-time is the most basic space-time model which is homogeneous and anisotropic and has been extensively used to study the impact of anisotropy on cosmic evolution [11]. These models are more abstract than the Friedmann-Lemaître-Robertson-Walker (FLRW) universe, and help to understand the transition between anisotropy and isotropy.

In the present work, the study of cosmological models based on the variable gravitational constant G and cosmological constant Λ in the framework of the General Relativity theory is continued. In f(R,T) gravity, we assume that we have a homogeneous and anisotropic Bianchi Type I universe in which a perfect fluid is present. The exact functional form is f(R,T) = R + 2f(T)

If is adopted, there are corrections to Einstein's field equations, due to the coupling between the energy-momentum tensor and the geometry of space-time [8].

By making the assumption that the cosmological constant is time dependent with a relationship, Exact solutions of the modified field equations can be obtained.

$\Lambda \propto R^{-2}$,

where R is the mean scale factor of the universe. The resulting model is able to describe both the inflationary period of the early universe and the present-day accelerated expansion of the universe. A number of physical and geometrical parameters are explored, such as the Hubble parameter H, the gravitational constant G, the cosmological constant Λ, the expansion scalar θ, and the shear scalar σ^2.

The analysis shows that the evolution of the universe is consistent and there is a coupling between the shear scalar, the gravitational constant, the cosmological constant and the Hubble parameter. The model has a singularity of the point type at the initial epoch where Λ has the value of infinity. The gravitational constant changes dynamically with cosmic time and the cosmological constant decreases as the universe expands. Moreover, as the universe evolves, the anisotropy decreases and at late times the universe tends to be isotropic ($\sigma/\theta \rightarrow 0$). No explicit equation of state or a pre-given functional form of the G has been imposed, and the solutions thus obtained are more general and physically realistic.

Hence, the present model constitutes a viable idea for exploring cosmological evolution in anisotropic space-time in the framework of modified f(R,T) gravity and it also plays a role in understanding how the variation of the gravitational and cosmological parameters can affect the evolution of the universe.

Metric and Field Equations

The line element for Bianchi type *I* space-time is given by,

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \tag{1}$$

The cosmological constant is invoked to explain the accelerated stage of the expansion of the universe. The EFE of $f(R,T)$ theory of gravity for this model for the function $f(R,T) = R + 2f(T)$ is,

$$R_{ij} - \frac{1}{2}g_{ij}R = [-8\pi + 2f'(t)]G(t)T_{ij} - \Lambda(t)g_{ij} + [2Pf'(t) + f(t)]g_{ij}, \tag{2}$$

Where the symbols involving in the above equation have their usual meaning. Phenomenological solution of so called "cosmological constant problem" is obtained by considering a time dependent cosmological constant. The time variation of cosmological constant Λ and Newtanion gravitational constant G are closely associated to each other in the classical models and therefore time variation of both G and Λ can be taken simultaneously in the field equations. We consider that

$f(T) = \lambda T$ where λ is a constant and T is the trace of the energy tensor of matter source. Eq. (2) can be written as,

$$R_{ij} - \frac{1}{2} g_{ij} R = [-8\pi + 2\lambda] G(t) T_{ij} + \Lambda(t) g_{ij} + (\rho - q)\lambda g_{ij}, \tag{3}$$

We assume that comic matter is taken is to be perfect fluid given by the energy momentum tensor

$$T_{ij} = (p + q) v_i v_j + g_{ij}, \tag{4}$$

where ρ and q are isotropic pressure and energy density of the fluid. We take of state

$$P = w\rho \quad 0 \le w \le 1, \tag{5}$$

v^i is the four velocity vector of the fluid satisfying

$$g_{ij} v_i v_j = -1. \tag{6}$$

For the metric Eq. (1) and energy-momentum tensor Eq. (4) in commoving system of coordinates, the field equation Eq. (3) yields

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = [-8\pi + 2\lambda] G(t)\rho - \lambda(\rho - p) - \Lambda(t), \tag{7}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = [8\pi - 2\lambda] G(t)\rho - \lambda(\rho - p) - \Lambda(t), \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = [8\pi - 2\lambda] G(t)\rho - \lambda(\rho - p) - \Lambda(t), \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = [8\pi - 2\lambda] G(t)\rho - \lambda(\rho - p) - \Lambda(t). \tag{10}$$

In view of vanishing of the divergence of Einstein tensor, we have

$$(-8\pi + 2\lambda) G \left[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right] + (-8\pi + 2\lambda)\rho \dot{G} - \dot{\rho}\lambda - \dot{\Lambda} = 0. \tag{11}$$

The usual energy conversion equation of general relativity quantities is

$$(-8\pi + 2\lambda) G \left[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right] = 0. \tag{12}$$

Eq. (11) together with Eq. (12) puts G and Λ in some short of coupled field given by

$$(-8\pi + 2\lambda)\rho \dot{G} - \rho\lambda - \dot{\Lambda} = 0. \tag{13}$$

Implying that G is constant whenever Λ is constant. Using Eq. (5) in Eq. (12) and integrating, we get $k > 0$

$$\rho = \frac{k}{R^2}. \tag{14}$$

we define, R as average scale factor of Bianchi type-1 universe.

$$R = (ABC)^{\frac{1}{3}}. \tag{15}$$

The Hubble parameter H, volume expansion θ, shear σ and deceleration q are given by

$$\theta = 3H = \frac{3\dot{R}}{R}, \quad \sigma = \frac{k}{\sqrt{3R^3}}, \quad k > 0 \text{ (Constant)}$$

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2}$$

Einstein field's Eq. (7-10) can also be written in terms of Hubble parameter H, shear σ and deceleration q as

$$H^2(2q + 1) - \sigma^2 = [8\pi - 2\lambda] G(t)\rho - \lambda(\rho - p) - \Lambda(t) \tag{16}$$

$$3H^2 - \sigma^2 = [8\pi - 2\lambda] G(t)\rho - \lambda(\rho - p) + \Lambda(t). \tag{17}$$

On integrating Eq. (7-10), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{R^3}, \tag{18}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{R^3}, \tag{19}$$

where k_1 and k_2 are constants of integration. From Eq. (17), we obtain

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{24\pi G\rho}{\theta^2} - \frac{3\Lambda}{\theta^2} + \frac{6\lambda G\rho}{\theta^2} + \frac{3\lambda(\rho - p)}{\theta^2}. \tag{20}$$

Implying that $\Lambda \ge 0$

$$0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}, \quad 0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3} \tag{21}$$

Thus the presence of positive Λ lower the upper limit of anisotropy where as a negative Λ contributes to the anisotropy. Eq. (20) can be written as

$$\frac{\sigma^2}{3H^2} = 1 - \frac{8\pi G\rho}{3H^2} - \frac{\Lambda}{3H^2} + \frac{2\lambda G\rho}{3H^2} + \frac{\lambda(\rho - p)}{3H^2} = 1 - \frac{\rho}{\rho_c} - \frac{\rho_v}{\rho_c} + \frac{2\lambda G\rho}{3H^2} + \frac{\lambda(\rho - p)}{3H^2}, \tag{22}$$

where $\rho_c = \frac{3H^2}{8\pi G}$ is critical density and $\rho_v = \frac{\Lambda}{8\pi G}$ is the vacuum density. Form Eq. (16-17) we get

$$\frac{d\theta}{dt} = (12\pi - 3\lambda) G(p - \rho) - H^2 - 3\Lambda. \tag{23}$$

Thus the universe will be decelerating phase for negative Λ and for positive Λ universe will be slow down the rate decrease, showing that the rate volume expansion decreases during time evaluation and presence of positive Λ, slows down the rate of this decreases whereas a Λ negative would promote it.

Solution of the Filed Equations

The system of equations Eq. (7, 10 and 13) supply only five equations in seven unknown parameters $(A, B, C, \rho, p, \Lambda, G)$. Two extra equations are needed to solve the system completely. For this purpose we take cosmological term in proportional to R^{-2}, where 'a' is a positive constant i.e., we take decaying vacuum energy density

$$i.e \quad \Lambda = \frac{a}{R^2}. \tag{24}$$

This variation law was proposed by Olson, Pavon, Maia, Silveria and Torres. Because observations suggest that Λ is very small in the present universe, a decreasing functional from permits Λ to be large in early universe.

Using Eq. (14) and Eq. (24) in Eq. (13) we get

$$G = \frac{1}{[4\Pi - \lambda]}\left[\left(\lambda + \frac{a}{k}\right) lnR\right]. \tag{25}$$

From Eq. (16, 17, 24, and 25) we get

$$\frac{\ddot{R}}{R} + \left[\frac{\dot{R}}{R}\right]^2 - \frac{a}{R^2} = 0 \tag{26}$$

Find the time evolution of Hubble parameter, integrating Eq. (26) we get

$$\frac{\dot{R}}{R} = H = \sqrt{a}\left[\sqrt{a}t + t_0\right]^{-1}, \tag{27}$$

where t_0 is a constant of integration. The integration constant is related to the choice of origin of time.

From the Eq. (27), we get the scale factor

$$R = \left[\sqrt{a}t + t_0\right]. \tag{28}$$

From the Eq. (28) in Eq. (18 and 19) in the metric Eq. (1), we get

$$ds^2 = -dt^2 + R^2\left[m_1^2 exp\left\{\left[\frac{-1}{\sqrt{a}R^2}\right]\left[\frac{2k_1+k_2}{3}\right]\right\}\right]dx^2 + R^2\left[m_2^2 exp\left\{\left[\frac{-1}{\sqrt{a}R^2}\right]\left[\frac{2k_2+k_1}{3}\right]\right\}\right]dx^2 + R^2\left[m_3^2 exp\left\{\left[\frac{-1}{\sqrt{a}R^2}\right]\left[\frac{-2k_2+k_1}{3}\right]\right\}\right]dz^2, \tag{29}$$

where $m_1, m_2,$ and m_3 are constant. For the model (29) the spatial V, density ρ gravitational constant G and cosmological constant Λ are

$$V = R^3 = \left[\sqrt{a}t + t_0\right]^3, \tag{30}$$

$$\rho = \frac{k}{\left[\sqrt{a}t+t_0\right]^2}, \tag{31}$$

$$G = \frac{1}{[4\Pi - \lambda]}\left[\left(\lambda + \frac{a}{k}\right) ln\left(\sqrt{a}t + t_0\right)\right], \tag{32}$$

$$\Lambda = \frac{a}{\left[\sqrt{a}t+t_0\right]^2}, \tag{33}$$

Expansion scalar θ and shear σ

$$\theta = 3\sqrt{a}\left[\sqrt{a}t + t_0\right]^{-1}, \tag{34}$$

$$\sigma = \frac{k}{\sqrt{3}\left[\sqrt{a}t+t_0\right]^3}. \quad k > 0 \tag{35}$$

Summary and Conclusion

In this section, I summarize the results obtained in this research. Here, my work is extension of Cosmological Model With varying G and Λ in general relativity. A lot of work has been done by Saha (2005), in studying the anisotropic Bianch type I Cosmological Model in general relativity with varying G and Λ. In this article, I extend this work by converting cosmological Model with homogenous Bianchi type I space time with variable G and Λ containing matter in the form of perfect fluid inn the framework of $f(R, T)$gravity.

In this article, I concentrated on the first class as $f(R, T) = R + 2f(T)$. We considered the modified gravity which naturally unifies two expansion phases of the universe i.e., inflation at early time and cosmic acceleration at current approach. The model based on the exact solution of the $f(R, T)$ gravity field equations for the anisotropic Biandi-I space-time filled with perfect fluid with time dependent Λ term. Solution of Einstein's equations involves the conservation of energy-momentum tensor. Through physical process I detected the evolution of different parameters which imply that dissimilar parameters are coupled. In this manner the field equation provide trivial assumption implying a linear linkage among σ^2, G, Λ and H.

In short, I have explored Bianchi type I space-time in variable G and Λ in presence of perfect fluid with Cosmological term Λ proportional to R^{-2} (where R is scale factor). Initially the model has a point type singularity, gravitational constant $G(t)$ is decreasing and Cosmological constant Λ is infinite at this time, when time increases Λ decreases. Here, I have neither assumed equation of state not particular form of G. The model approach isotropy for large value of t, the model is quasi-isotopic i.e., $\frac{\sigma}{\theta} = 0$.

References

1. Einstein A. The foundation of the general theory of relativity. Ann Phys. 1916.
2. Riess AG, Filippenko AV, Challis P, Clocchiatti A, Diercks A, Garnavich PM, *et al*. Observational evidence from supernovae for an accelerating universe. Astron J. 1998;116:1009-38.
3. Perlmutter S, Aldering G, Goldhaber G, Knop RA, Nugent P, Castro PG, *et al*. Measurements of Ω and Λ from high-redshift supernovae. Astrophys J. 1999;517:565-86.
4. Planck Collaboration. Planck 2018 results. VI. Cosmological parameters. Astron Astrophys. 2020;641:A6.
5. Weinberg S. The cosmological constant problem. Rev Mod Phys. 1989;61(1):1-23.

6. Starobinsky AA. A new type of isotropic cosmological models without singularity. Phys Lett B. 1980;91(1):99-102.
7. De Felice A, Tsujikawa S. f(R) theories. Living Rev Relativ. 2010;13:3.
8. Harko T, Lobo FSN, Nojiri S, Odintsov SD. f(R,T) gravity. Phys Rev D. 2011;84(2):024020.
9. Dirac PAM. The cosmological constants. Nature. 1937;139:323.
10. Saha B. Anisotropic cosmological models with variable G and Λ. Astrophys Space Sci. 2005;299:149-58.
11. Ellis GFR, MacCallum MAH. A class of homogeneous cosmological models. Commun Math Phys. 1969;12:108-41.

How to Cite This Article

Ahmad S. A generalized Bianchi Type I cosmological model with time-varying gravitational and cosmological constants in f(R,T) gravity. Int J Eng Comput Appl. 2026;2(4):27-31. doi:10.54660/IJECA.2026.2.4.27-31.